Information Loss and Entanglement in Quantum Theory

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Quantum Theory somewhat comparable to construction of *Babylonian tower*, with similar consequences …!

Claims:

(1) Without *information loss* and *entanglement* retrieval of information from quantum systems by measurements/ observations would be *impossible*; no info. paradoxes!

(2) Operator algebras (including type III_1 factors!) have been invented to be used in QT, rather than to be ignored!

Credits and Contents:

Motivation from recent experiments, in particular the ones of the *Haroche-Raimond* group; papers by *Bauer & Bernard*, *Maassen & Kümmerer*, and others; joint work with my former PhD student *Baptiste Schubnel*; (some joint efforts with *Ballesteros, Faupin, Fraas, Pickl, Schilling, Schubnel*); discussions with Detlev Buchholz and others.

1. *Introduction*
2. *Information Loss*
3. *Projective (von Neumann) measurements*
4. *Conclusions*

1. Introduction – Questions to be Addressed

In our courses, we tend to describe quantum-mechanical systems as pairs of a Hilbert space, $\mathcal{H}$, and a propagator, $U(t,s)$, describing time-evolution. Unfortunately, these data encode almost *no invariant structure* (besides spectral properties of $U(t,s)$) and give the erroneous impression that quantum theory might be deterministic. Thus, among *fundamental problems of quantum theory* are:

- What do we have to add to the usual formalism of quantum theory in order to arrive at a mathematical structure that (through "interpretation") can be given physical meaning, *independently of "observers"*?

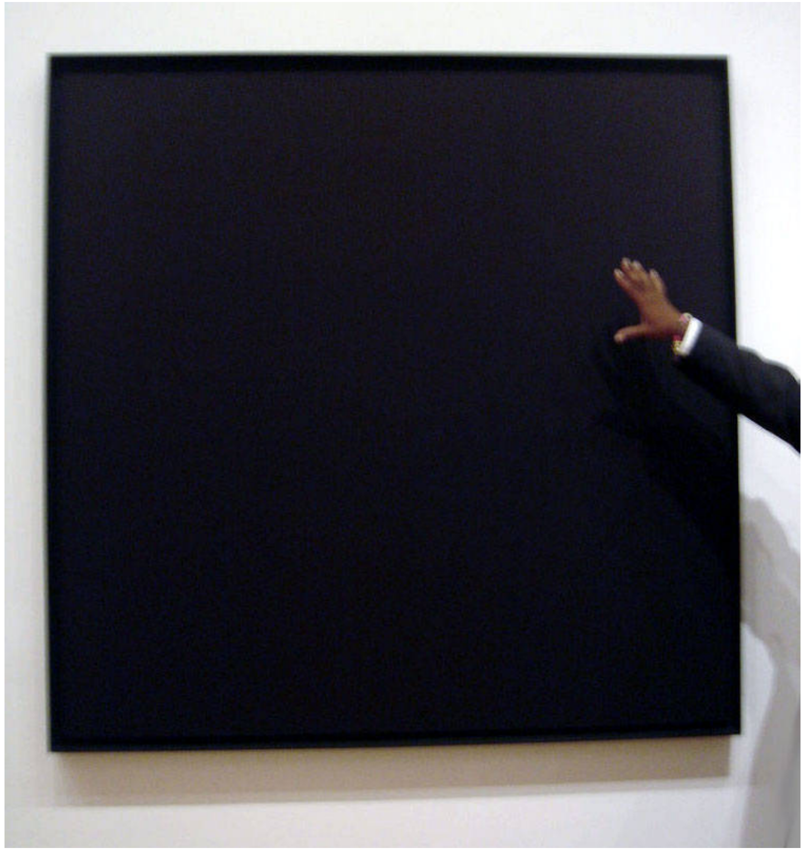
Questions, ctd.

- Where does *intrinsic randomness* in quantum theory come from, given the deterministic character of the Schrödinger equation? In which way does it differ from classical randomness?
- Do we understand the *effective dynamics* of quantum systems, e.g., ones in contact with a heat bath (*“Quantum Brownian Motion”*), or under repeated measurements (*“quantum jumps and tracks”*)?
- What do we mean by an *“isolated system”* in quantum mechanics, and why is this an important notion? How does one *prepare* a system in a *prescribed state*?

Rescue & answers come from

Information Loss & Entanglement

*QM is QM-as-QM and everything else is everything else**



* “The one thing to say about art is that it is one thing. Art is art-as-art and everything else is everything else.”

Ad Reinhardt

2. Information Loss

A simple-minded definition of quantum-mechanical systems:

An *isolated* quantum system, *S*, is characterized by following choices:

(i) $(\mathcal{H}, U(t,s)), \mathbb{R} \ni t, s$ (U = unitary propagator)

(ii) a list, $\mathcal{O}_S = \{a_i\}_{i \in I_S}$, of bounded, selfadjoint operators on $\mathcal{H}$ representing *physical quantities/ potential properties* of *S* that can be measured in direct *"projective measurements"*;

(*S* must be an *isolated* system *chosen large enough* for quantities represented by $a_i, i \in I_S$, to be measureable).

Information Loss, ctd.

Choose fiducial time, t_0, and define (Heisenberg picture)

$$a(t) := U(t_0, t)aU(t, t_0), \qquad a \in \mathcal{O}_S,$$

to be the operator representing the pot. prop. corresp. to $a \in \mathcal{O}_S$ at time t ; ➔ list of ops. $\mathcal{O}_S(t)$

Pot. properties, $a(s)$, measureable/observable at times $s \geq t$ generate a W*alg. $\mathcal{E}_{\geq t}$:

$$\mathcal{E}_{\geq t} := \langle \sum \prod_i a_i(t_i) | a_i \in \mathcal{O}_S, t_i \geq t \rangle^{-}$$

$$\mathcal{A}_S := \mathcal{E}_{\geq -\infty}, \qquad \mathcal{S}_S \text{ (states)} \tag{1}$$

$$B(\mathcal{H}) \supseteq \mathcal{A}_S \supseteq \mathcal{E}_{\geq t} \supseteq \mathcal{E}_{\geq s} \supseteq \mathcal{O}_S(s), \; s \geq t \tag{2}$$

$\neq$ ← *Information Loss!*

Information Loss, ctd.

Define $$\tau_s(a(t)) := a(s+t)$$

so that $$\tau_s : \mathcal{E}_{\geq t} \to \mathcal{E}_{\geq (t+s)}$$

τ_s is a *endom; τ_s *not* a *autom $\Leftrightarrow$ *information loss*

$\Rightarrow$ *entanglement with "lost" degrees of freedom!*

It is easy to construct examples of (generally *non-autonomous*) quantum-mechanical systems exhibiting *information loss*, in the sense of Eq. (2):

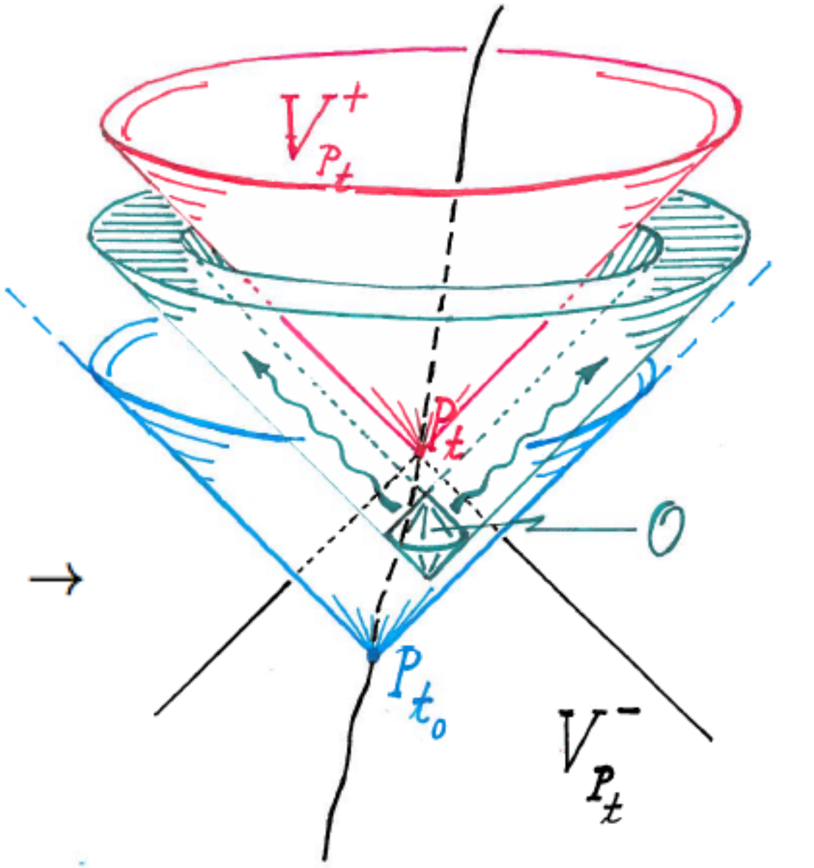
(1) Independent "probes", E_j, j =1,2,3,..., with E_j being destroyed at time τ j, (τ discrete time step)

(2) "Small" systems *temporarily* interacting with quantized wave medium, (e.g., photons, phonons, etc.)

(3) *Information loss in theories like QED*

All ops. in $\mathcal{E}_{\geq t}$
are localized
in $V^+_{P_t}$, $P_t = (t, \vec{x})$

$\mathcal{E}_{\geq t_0} \supset \mathcal{E}_{\geq t}$,
for $t > t_0$
$(\mathcal{E}_{\geq t})' \cap \mathcal{E}_{\geq t_0} \supset \mathcal{A}^{\text{out}}_{\mathcal{O}}$ $\rightarrow$



worldline of *JF*

3. Projective (von Neumann) measurements

In this section, we discuss how phys. quantities are measured.

Some fundamental questions to be addressed:

(1) What is meant by a "measurement" of $a \in \mathcal{O}_S$? Around which time t does it take place ? A measurement of a ought to result in "a having a value", i.e., become an "empirical/objective property" of *S*

$\Leftrightarrow$ state on $\mathcal{E}_{\geq t} \simeq$ incoherent mixture of eigenstates of $a(t)$,

at some time *t*. *(Proj. measnts. vs. indirect (Kraus) measnts.)*

(2) Given a state of *S*, does *QM predict* which $a \in \mathcal{O}_S$ will be measured first; what does *QM* predict about the outcome of measnt. of a ? In which way is *QM* intrinsically *indeterministic*? Why does a measnt. of a have a *random outcome*?

Projective measurements, ctd.

Projective measurements

We have to clarify what may be meant by a *projective measurement* of a potential property $a \in \mathcal{O}_S$ and what the role of *information loss & entanglement* in measnts. is:

$$\mathcal{O}_S \ni a = a^* \quad \text{with eigenvalues} \quad \alpha_1, \alpha_2, \ldots, \alpha_k,$$

$$a(t) = \sum_{j=1}^{k} \alpha_j \Pi_j(t) \qquad (3)$$

"Measurement/observation" of a around time *t*

$\Leftrightarrow$ a is an *"empirical/objective property"* of *S* around time *t* :

Projective measurements, ctd.

State ρ is an *incoherent superposition* of eigenstates of a:

$$\rho(b) = \sum_{j=1}^{k} \rho(\Pi_j(t) b \Pi_j(t)), \qquad \text{for all } b \in \mathcal{E}_{\geq t} \quad (4)$$

where ρ is the state of S right before measnt. of a, i.e.,

$$\rho = \rho_t := \rho|_{\mathcal{E}_{\geq t}} \qquad (5)$$

Information Loss $\Rightarrow \rho = \rho_t$ is usually a mixed state on $\mathcal{E}_{\geq t}$ *even* if the initial state of S has been *pure*, as a state on $\mathcal{A}_S$!

Projective measurements, ctd.

Suppose, for simplicity, that $\mathcal{E}_{\geq t}$ is isomorphic to some $B(\mathcal{H}_t)$, (i.e., $\mathcal{E}_{\geq t}$ is of type $I_\infty \Rightarrow$ syst. *non-autonomous*)

$$\text{Eq. (4)} \quad \Leftrightarrow \quad [a(t), P_t] = 0, \qquad (6)$$

where P_t is the density matrix on $\mathcal{E}_{\geq t}$ corresp. to ρ_t

<u>*Definition*</u>

$a \in \mathcal{O}_S$ is measured/observed around time $t \Leftrightarrow a$ is an *"empirical/objective prop."* of *S* around time *t* iff

$$a(t)|_{RangeP_t} \approx_t F(P_t) \cdot z, \text{ for some bd. fu. } F, \qquad (7)$$

and some z in the center of $\mathcal{E}_{\geq t}$. More generally, $a(t)$ belongs to the *center of the centralizer of the state* ρ_t , $\mathcal{Z}_{\geq t}$.
Note that R.S. of (7) is *cond.expectation* of $a(t)$ w.r. to $\mathcal{Z}_{\geq t}$. Eq. (7) $\Rightarrow$ Eq. (4)! ➚ <u>Tomita-Takesaki theory!</u>)

Projective measurements, ctd.

Axiom A

If a is measured (i.e., an empirical/objective prop. of *S*) around time t then a has a *value* $\in \{\alpha_1, \dots, \alpha_k\}$ around time t.

The value α_j of a is observed w. probability

$$p_j(t) = \rho(\Pi_j(t)) \tag{8}$$

If α_j is observed around time t then the state

$$\rho_j^a(\cdot) := p_j(t)^{-1} \cdot \rho(\Pi_j(t)(\cdot)\Pi_j(t)) \text{ on } \mathcal{E}_{\geq t} \tag{9}$$

should be used for improved predictions of future after time t. (Eq. (8) is *Born's Rule*, eq. (9) *"collapse postulate"*.)

Projective measurements – *summary*

(1) Given the *initial state* of the system *S*, *time evolution*, {*U(t,s)*}, *determines* which pot. prop. $a \in \mathcal{O}_S$ will first become empirical (objective, measureable), and around which time!

(2) Measnt. of a_2 is *independent* of an *earlier* measnt. of a_1 iff a_2 becomes empirical/objective *after* time of measnt. of a_1, no matter what the outcome of measnt. of a_1 was, i.e., *for all states* $\rho_j^{a_1}(\cdot)$ *, j= 1, … , k,* with $\rho_j^{a_1}(\cdot)$ as in (9).

$\Rightarrow$ *Decoherence, "consistent histories".*

(3) Time of measurement*:* Time, t_*, of observation of a det. by minimizing in t the fu. $\| a(t)|_{RangeP_t} - F(P_t) \cdot z \|$, where $F(P_t) \cdot z$ is the "*cond. exp.*" of $a(t)$ onto $\mathcal{Z}_{\geq t}$.

(4) General theory of repeated measurements: "POVM's".

Projective measurements – *summary*

(5) A state is called *"passive"* iff the center, $\mathcal{Z}_{\geq t}$, of the centralizer of $\mathcal{E}_{\geq t}$ is *time-independent*. There are plenty of examples of passive states:

- Equilibrium (KMS) states at positive temperature in QFT; KMS states of a QFT in the space-time of a static black hole.
- Perturbations of the vacuum state by coherent clouds of massless particles (e.g., of photons).

Passive states have the property that they do *not admit any projective measurements/observations* of any physical quantities – besides measurements of *time-independent* parameters characteristic of the state in question, e.g., the temperature or a chemical potential of an equilibrium state, which, indeed, are time-independent quantities.

We have to learn more about which states and which types of time evolutions of isolated systems admit non-trivial measurements!

7. Conclusions

"In all my films, I have been faithful to these suspension points in the conclusions. Besides, I have never written the word 'END' on the screen."
(Federico Fellini)

"Everyone wants to understand art (physics). Why don't we try to understand the song of a bird? Why do we love the night, the flowers, everything around us, without trying to understand them? But in the case of a painting (result in physics), people think they have to understand." (Pablo Picasso)

Thank you for listening!